Reachability and deadlocking in multi-stage scheduling

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Gerhard Woeginger

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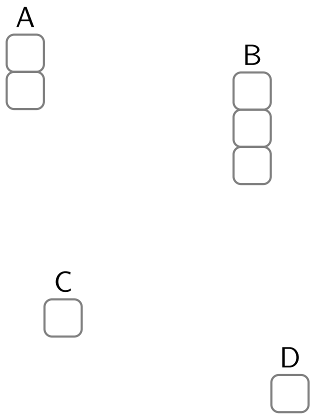
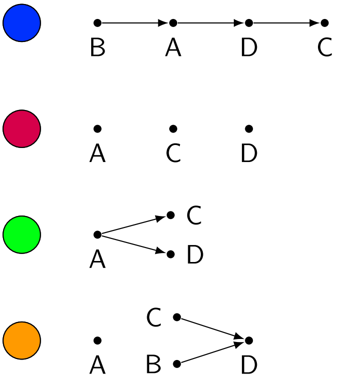
Scheduling systems

Machines



Jobs



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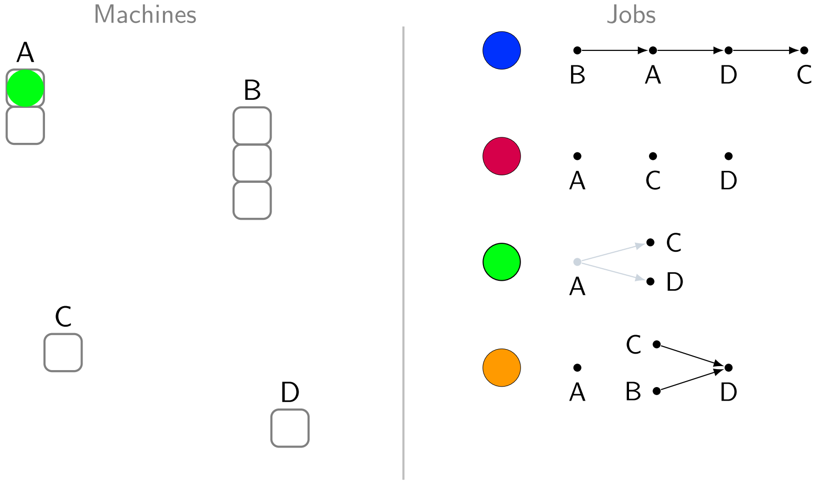

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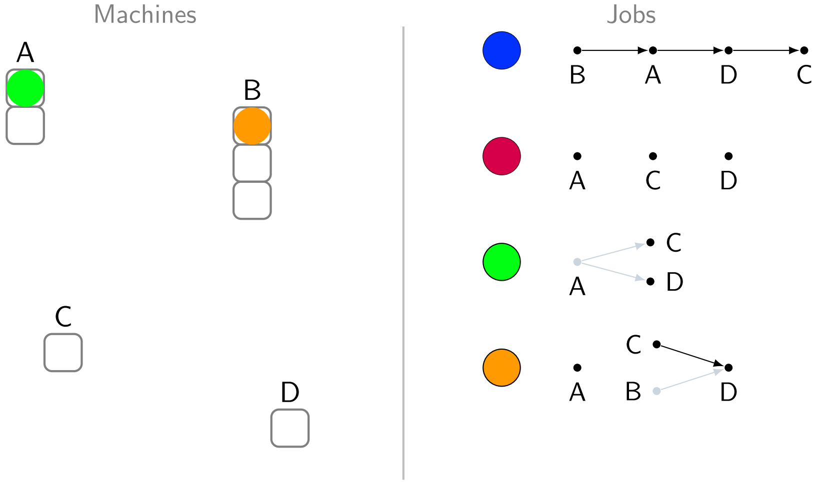

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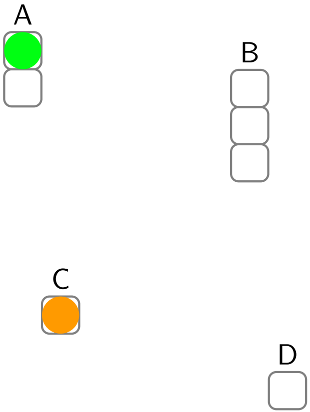

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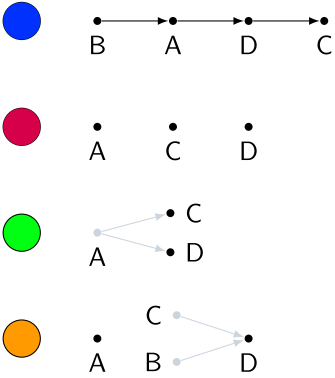

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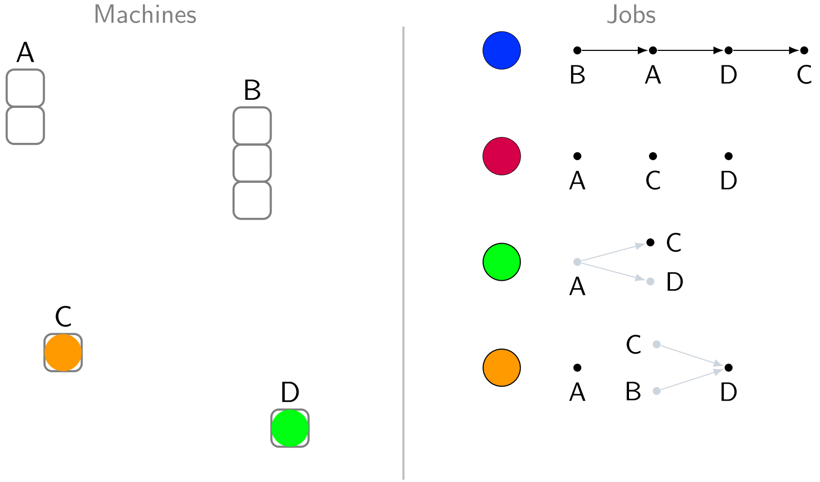

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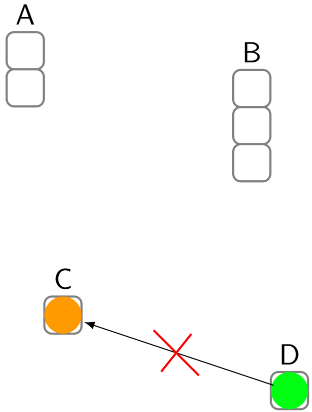

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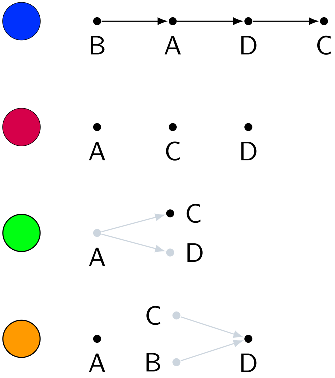

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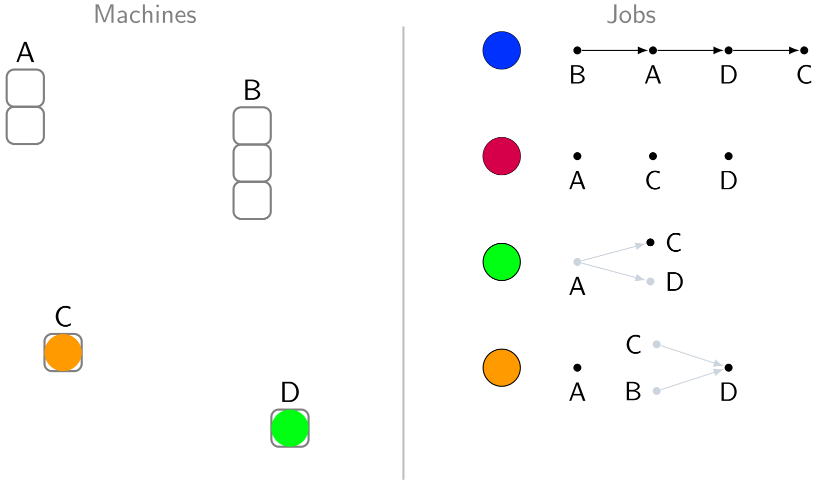

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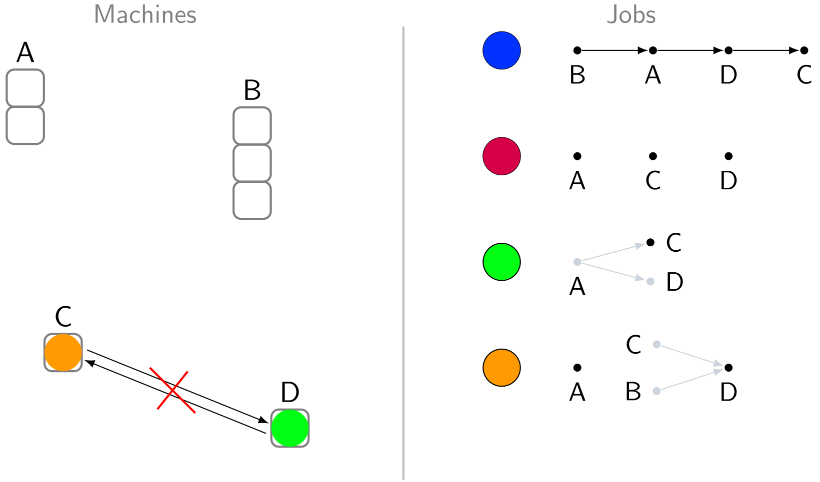
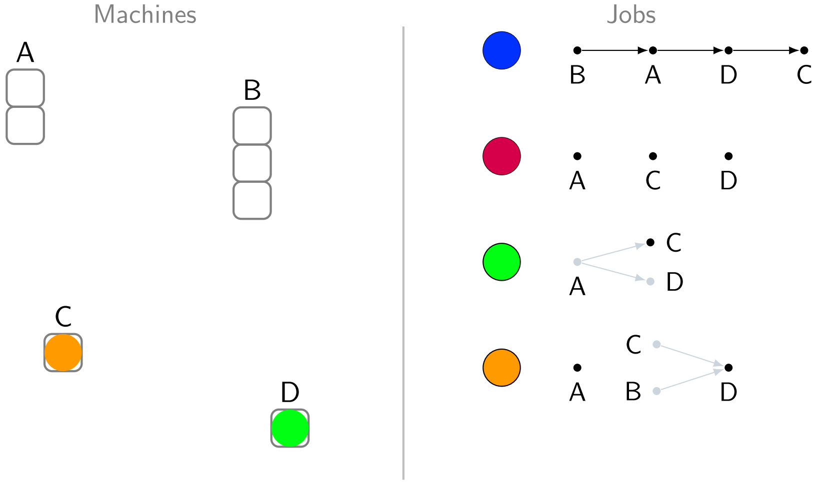


Unsynchronised job movements

Scheduling systems



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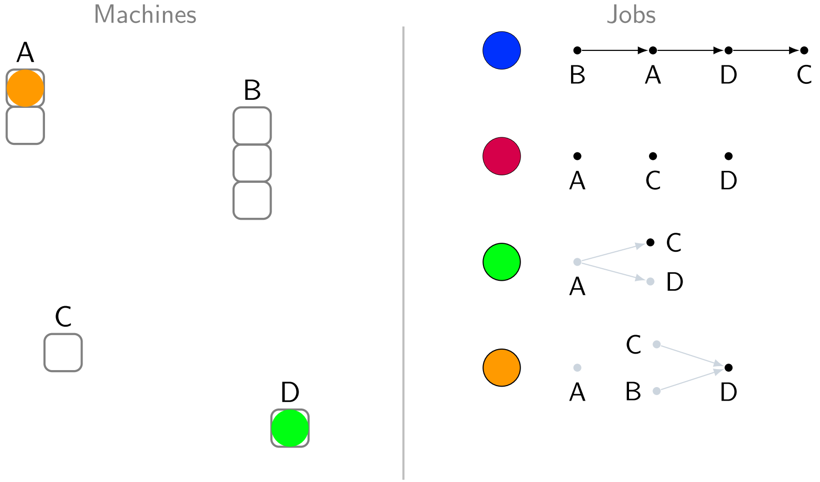

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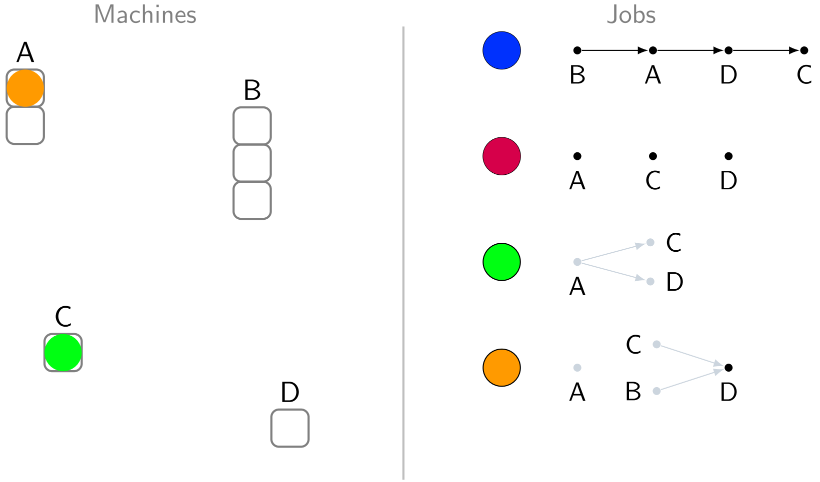

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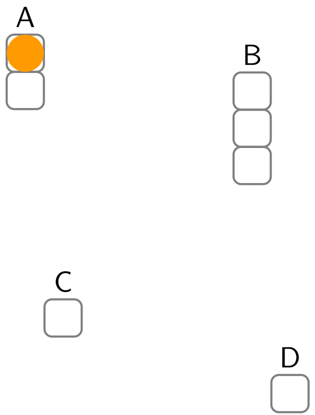

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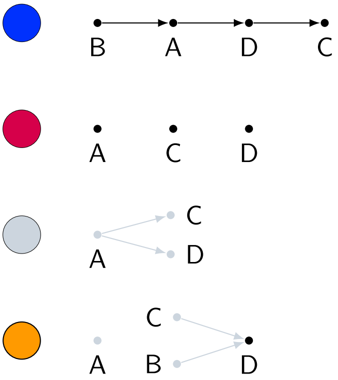

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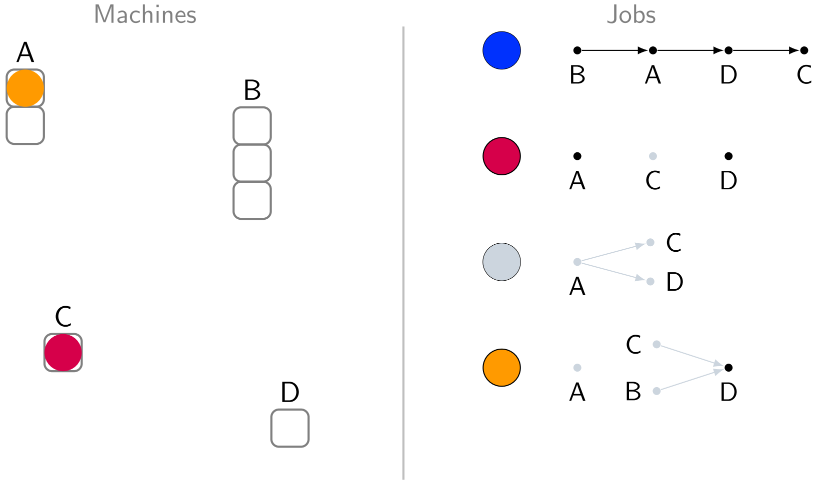

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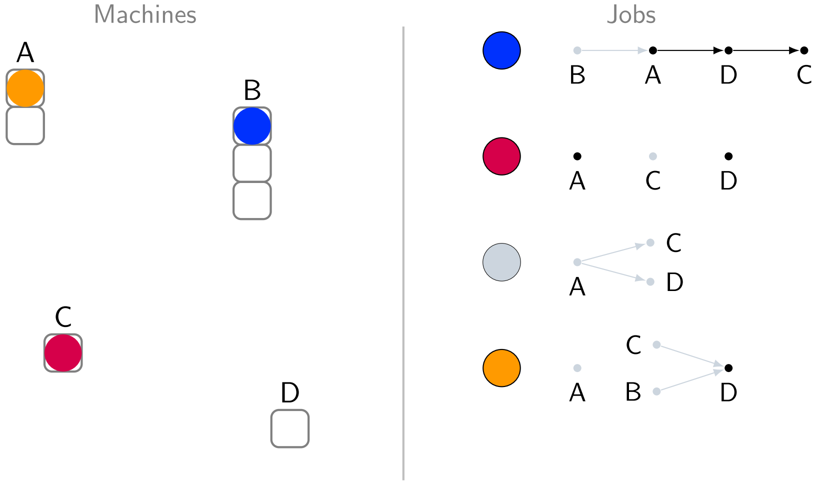

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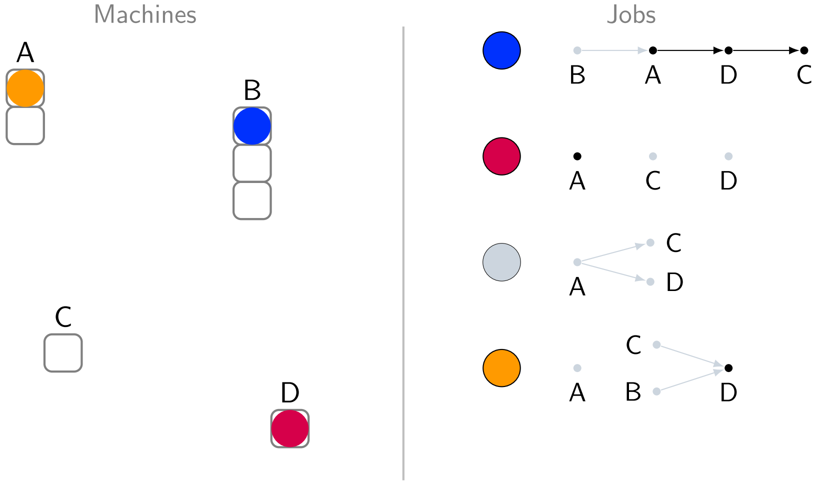

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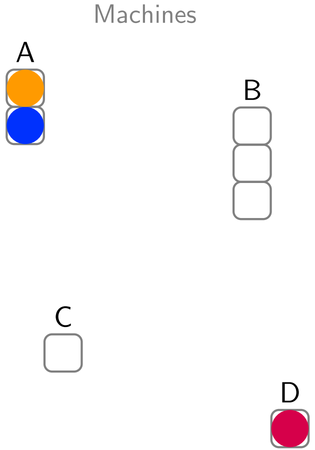

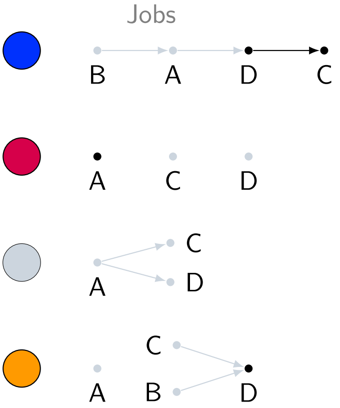

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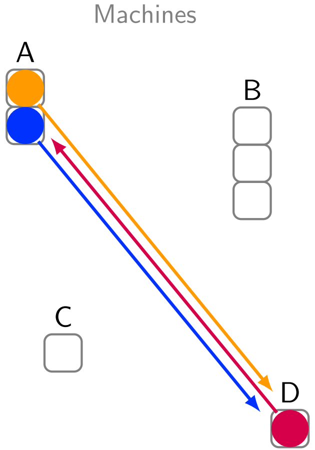
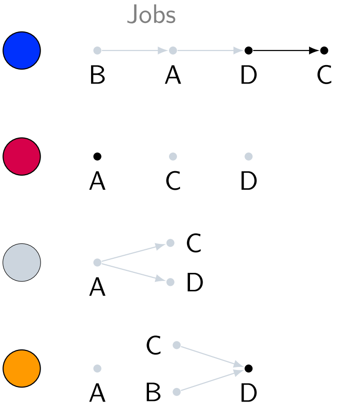
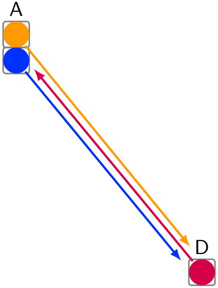




Basic Terms

- blocking set;
  - (sub-)set of machines $\mathcal{B} \neq \emptyset$ occupied to full capacity,
  - jobs on $\mathcal{B}$ need further processing, *next* only within $\mathcal{B}$



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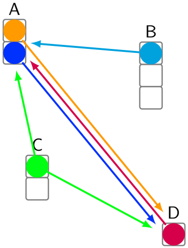

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Lemma

state contains blocking set $\implies$ *state unsafe*

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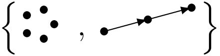
- scheduling system restricted to family of digraphs F;
  each job is isomorphic to digraph in F.



Previous Related Work

- M. Lawley and S. Reveliotis (2001). Deadlock avoidance for sequential resource allocation systems: hard and easy cases. *The International Journal of Flexible Manufacturing Systems* 13, 385–404.
- W. Sulistyono and M. Lawley (2001). Deadlock avoidance for manufacturing systems with partially ordered process plans. *IEEE Transactions on Robotics and Automation 17*, 819–832.
- C.E.J. Eggermont, A. Schrijver, G.J. Woeginger (2011). Analysis of multi-stage open shop processing systems. *International Symposium on Theoretical Aspects of Computer Science* (STACS), LIPIcs 9, 484–494.

Problems

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1. Recognize Safe State
   Instance : scheduling system, state s
   Question : Is state s safe?

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   Instance : scheduling system, state s
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   Instance : scheduling system
   Question : Can the system fall into a deadlock state?

Problems

How does restricting scheduling system affect complexity?

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Results

Results: 1. Safety

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Theorems (1996-2001)

For scheduling systems where either

(i) every machine capacity > 1 or

(ii) every job is unconstrained:

$$\text{state unsafe} \iff \text{state contains blocking set}$$

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Theorems (1996-2001)

For scheduling systems where either

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Lemma

Whether state contains blocking set is decidable in polynomial time

Results: 1. Safety

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Theorem

For scheduling system s.t. machines with capacity 1 occur only in unconstrained plans and out-stars:

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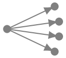

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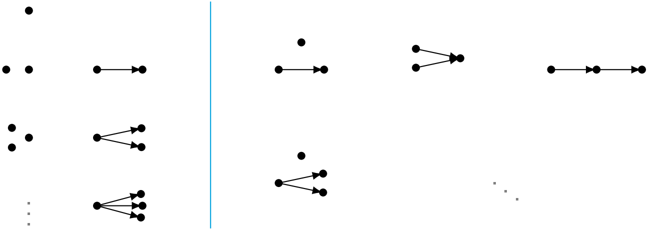
Theorem

RECOGNIZE SAFE STATE *NP-hard otherwise*



Results: 2. Reachability

REACHABILITY

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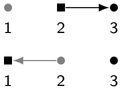
Example:

$\mathcal{M}$: $\{1, 2, 3\}$

J_1:

J_2:



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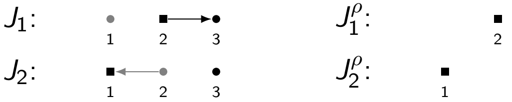

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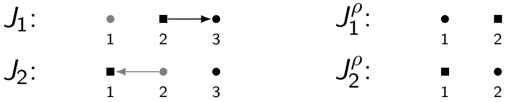

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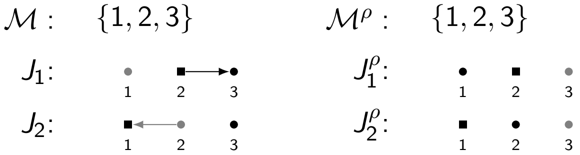

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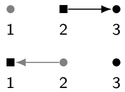

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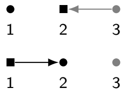

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Instance : scheduling system, state s
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Lemma

State s reachable $\iff$ state $\rho(s)$ is safe in new system

Results: 2. Reachability

REACHABILITY

Instance : scheduling system, state s

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Lemma

State s reachable $\Longleftrightarrow$ state $\rho(s)$ is safe in new system

Theorem

REACHABILITY *is decidable in polynomial time for scheduling system where machines with capacity 1 occur only in unconstrained plans and in-stars, and NP-hard otherwise.*

Results: 3. Deadlock

DEADLOCK

Instance : scheduling system

Question : Can the system fall into a deadlock?

Results: 3. Deadlock

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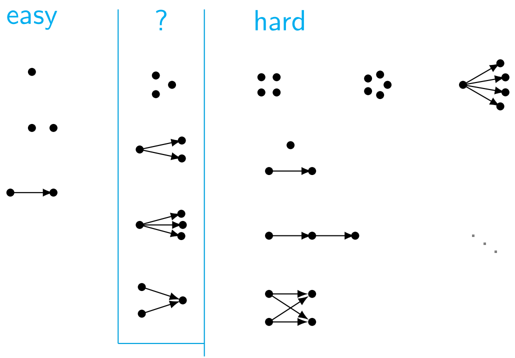

Proof: 1. Safety
sketch

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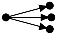
NOT ( or )

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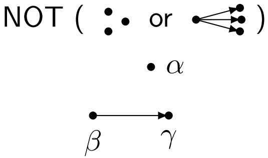

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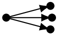
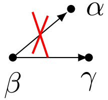
NOT ( or )



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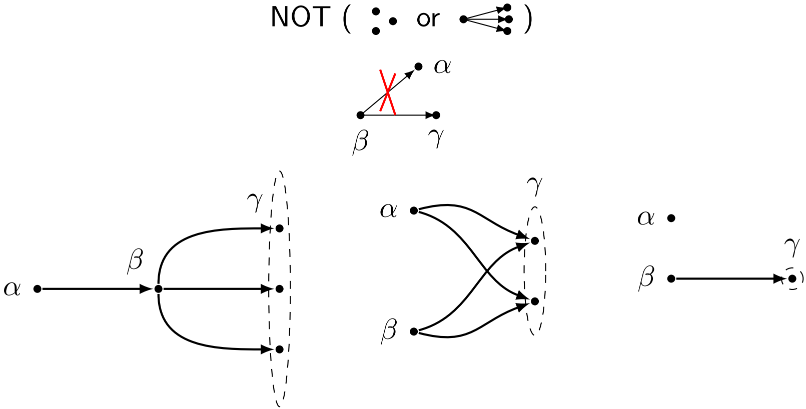

Decomposition: machine β s.t. set $\gamma := \text{succ}(\beta)$ minimal,
$\alpha \neq \beta$ sink on non-succesors, call rest δ.

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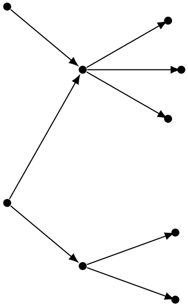

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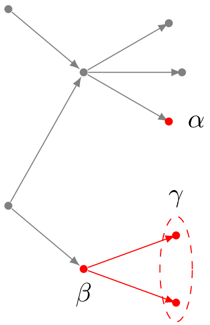

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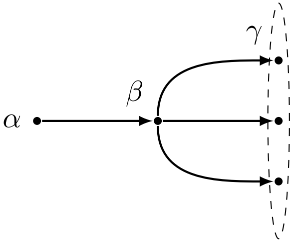

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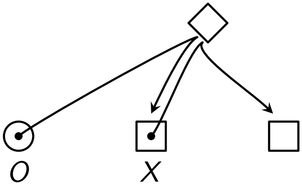

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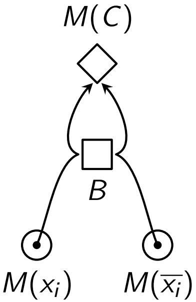

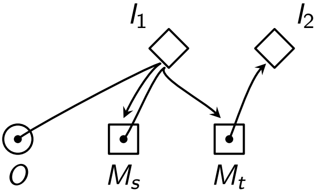

□ : internal machine
◇ : input machine
○ : output machine

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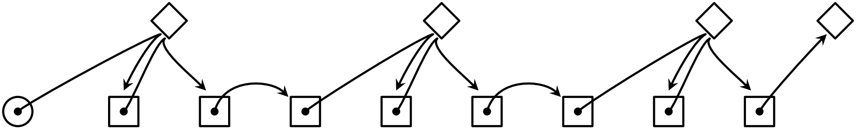

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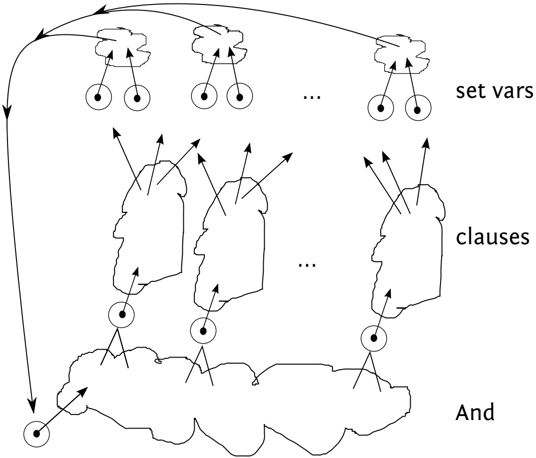

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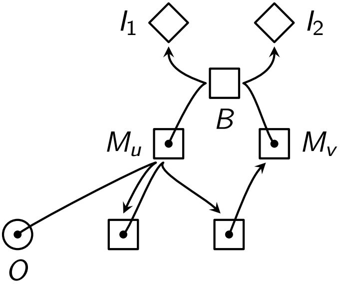

Questions / Comments ?